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On The Pair of Equations

$N_1 - N_2 = k, N_1 * N_2 = k^3 s^2, k \geq 0$ and square-free

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Abstract :

The thrust of this paper is to obtain many non-zero distinct integers N_1, N_2 such that $N_1 - N_2 = k, N_1 * N_2 = k^3 s^2, k \geq 0$ and square-free. A few numerical examples are given. Some observations among N_1, N_2 are presented .

Keywords : Integer pairs ,System of double Diophantine equations ,Diophantine problem

Introduction:

The theory of Diophantine equation is a treasure house in which the search for many hidden relations and properties form a treasure hunt. In fact, Diophantine problems dominated most of the celebrated unsolved mathematical problems. Certain diophantine problems come from physical problems or from immediate mathematical generalizations and others come from geometry in a variety of ways .Certain diophantine problems are neither trivial nor difficult to analyze. In this context ,one may refer [1-18].The above results motivated us to search for the integer solutions to some other choices of double diophantine equations. In this paper , many non-zero distinct integers N_1, N_2 such that $N_1 - N_2 = k, N_1 * N_2 = k^3 s^2, k \geq 0$ & square-free are obtained. A few numerical examples are given. Some observations among N_1, N_2 are presented .

Method of analysis:

Let N_1, N_2 be any two non-zero distinct integers such that

$$N_1 - N_2 = k, \tag{1}$$

$$N_1 * N_2 = k^3 * s^2 \qquad (2)$$

where $k \geq 0$ and square-free.

Eliminating N_2 between (1) and (2) , we have

$$N_1^2 - k N_1 - k^3 * s^2 = 0 \qquad (3)$$

Taking (3) as a quadratic in N_1 and solving for N_1, we have

$$N_1 = \frac{k(1 \pm \sqrt{4 * k * s^2 + 1})}{2} \qquad (4)$$

Let

$$y^2 = 4 * k * s^2 + 1 \qquad (5)$$

which is the well-known Pellian equation whose general solution (s_n, y_n) is given by

$$s_n = \frac{1}{2\sqrt{4k}} g_n, y_n = \frac{1}{2} f_n \qquad (6)$$

where

$$f_n = (y_0 + \sqrt{4k} s_0)^{n+1} + (y_0 - \sqrt{4k} s_0)^{n+1},$$
$$g_n = (y_0 + \sqrt{4k} s_0)^{n+1} - (y_0 - \sqrt{4k} s_0)^{n+1}$$

Considering the positive sign in (4) , the values of N_1 are given by

$$N_1 = N_1(k, n) = \frac{k(f_n + 2)}{4} \qquad (7)$$

and from (1) we have

$$N_2 = N_2(k, n) = \frac{k(f_n - 2)}{4} \qquad (8)$$

The recurrence relations satisfied by N_1, N_2 are respectively given by

$$N_1(k, n+2) - 2y_0 N_1(k, n+1) + N_1(k, n) = k(1 - y_0),$$
$$N_2(k, n+2) - 2y_0 N_2(k, n+1) + N_2(k, n) = k(y_0 - 1)$$

A few numerical examples are given in the Table below:

Table –Numerical examples

k	n	$N_1(k,n)$	$N_2(k,n)$
2	0	4	2
	1	18	16
	2	100	98
	3	578	576
3	0	12	9
	1	147	144
	2	2028	2025
	3	28227	28224
5	0	25	20
	1	405	400
	2	5*1445	5*1444
	3	5*25921	5*25920
	4	2325625	2325620

Observations:

(i) Each of the following expressions is a perfect square

(a) $\frac{4N_1(k,2n+1)}{k}$

(b) $\frac{4N_2(k,2n+1)}{k}+k$

(c) $\frac{2N_1(k,2n+1)+2N_2(k,2n+1)+2k}{k}$

(ii) Each of the following expressions is a cubical integer

(a) $\frac{4N_1(k,3n+2)+12N_1(k,n)}{k}-8$

(b) $\frac{4N_2(k,3n+2)+12N_2(k,n)}{k}+8$

(c) $\frac{2N_1(k,3n+2)+2N_2(k,n)+12N_1(k,n)-6k}{k}$

(d) $\frac{2N_1(k,3n+2)+2N_2(k,n)+12N_2(k,n)+6k}{k}$

(e) $\frac{32(N_1(k,n))^3+32(N_2(k,n))^3-48k^2N_1(k,n)+24k^3}{k^3}$

(f) $\frac{32(N_1(k,n))^3+32(N_2(k,n))^3-48k^2N_2(k,n)-24k^3}{k^3}$

(iii) Let

$Y=4N_1(k,n)-2k, X=2[N_1(k,n+1)-y_0*N_1(k,n)]+k(y_0-1)$

Then, (X, Y) satisfies the Hyperbola $k s_0{}^2 Y^2 - X^2 = 4k^3 s_0{}^2$

(iv) Let

$$Y = 4N_2(k,n) + 2k, X = 2[N_1(k,n+1) - y_0 * N_1(k,n)] + k(y_0 - 1)$$

Then, (X, Y) satisfies the Hyperbola $k s_0{}^2 Y^2 - X^2 = 4k^3 s_0{}^2$

(v) Let

$$Y = 4N_1(k,2n+1), X = 2[N_1(k,n+1) - y_0 * N_1(k,n)] + k(y_0 - 1)$$

Then, (X, Y) satisfies the Parabola $k^2 s_0{}^2 Y - X^2 = 4k^3 s_0{}^2$

(vi) Let

$$Y = 4N_2(k,2n+1) + 4k, X = 2[N_1(k,n+1) - y_0 * N_1(k,n)] + k(y_0 - 1)$$

Then, (X, Y) satisfies the Parabola $k^2 s_0{}^2 Y - X^2 = 4k^3 s_0{}^2$

(vii). Each of the following expressions is a bi-quadratic integer

(a) $\dfrac{4N_1(k,4n+3) + 16N_1(k,2n+1)}{k} - 4$

(b) $\dfrac{4N_2(k,4n+3) + 16N_2(k,2n+1)}{k} + 16$

(c) $\dfrac{4N_1(k,4n+3) - 4k}{k} + \dfrac{4[4N_1(k,n) - 2k]^2}{k^2}$

(d) $\dfrac{4N_2(k,4n+3)}{k} + \dfrac{4[4N_2(k,n) + 2k]^2}{k^2}$

(viii) Each of the following expressions is a quintic integer

(a) $\dfrac{4N_1(k,5n+4) - 20N_1(k,n)}{k} + 5\dfrac{(4N_1(k,n) - 2k)^3}{k^3} + 8$

(b) $\dfrac{4N_2(k,5n+4) - 20N_2(k,n)}{k} + 5\dfrac{(4N_2(k,n) + 2k)^3}{k^3} - 8$

(c) $\dfrac{4N_1(k,5n+4) - 20N_1(k,n)}{k} + \dfrac{20N_1(k,3n+2) + 60\,N_1(k,n)}{k} - 32$

(d) $\dfrac{4N_2(k,5n+4) - 20N_2(k,n)}{k} + \dfrac{20N_2(k,3n+2) + 60\,N_2(k,n)}{k} + 32$

(ix) $\dfrac{32(N_1(k,n))^3 - 32(N_2(k,n))^3}{k^3} - 8 = 24y_n{}^2$

Conclusion:

In this paper ,an attempt has been made to obtain many non-zero distinct integer solutions to the pair of diophantine equations $N_1 - N_2 = k, N_1 * N_2 = k^3 s^2, k \geq 0$ and square-free . It is worth to mention that, if N_1, N_2 are taken to represent the sides of a rectangle ,then ,its area is square multiple of the difference of its sides. The readers may search for other characterizations employing the system of double equations.

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